Potential Magnetic Fields:
Null-like Points and Squashing Factors

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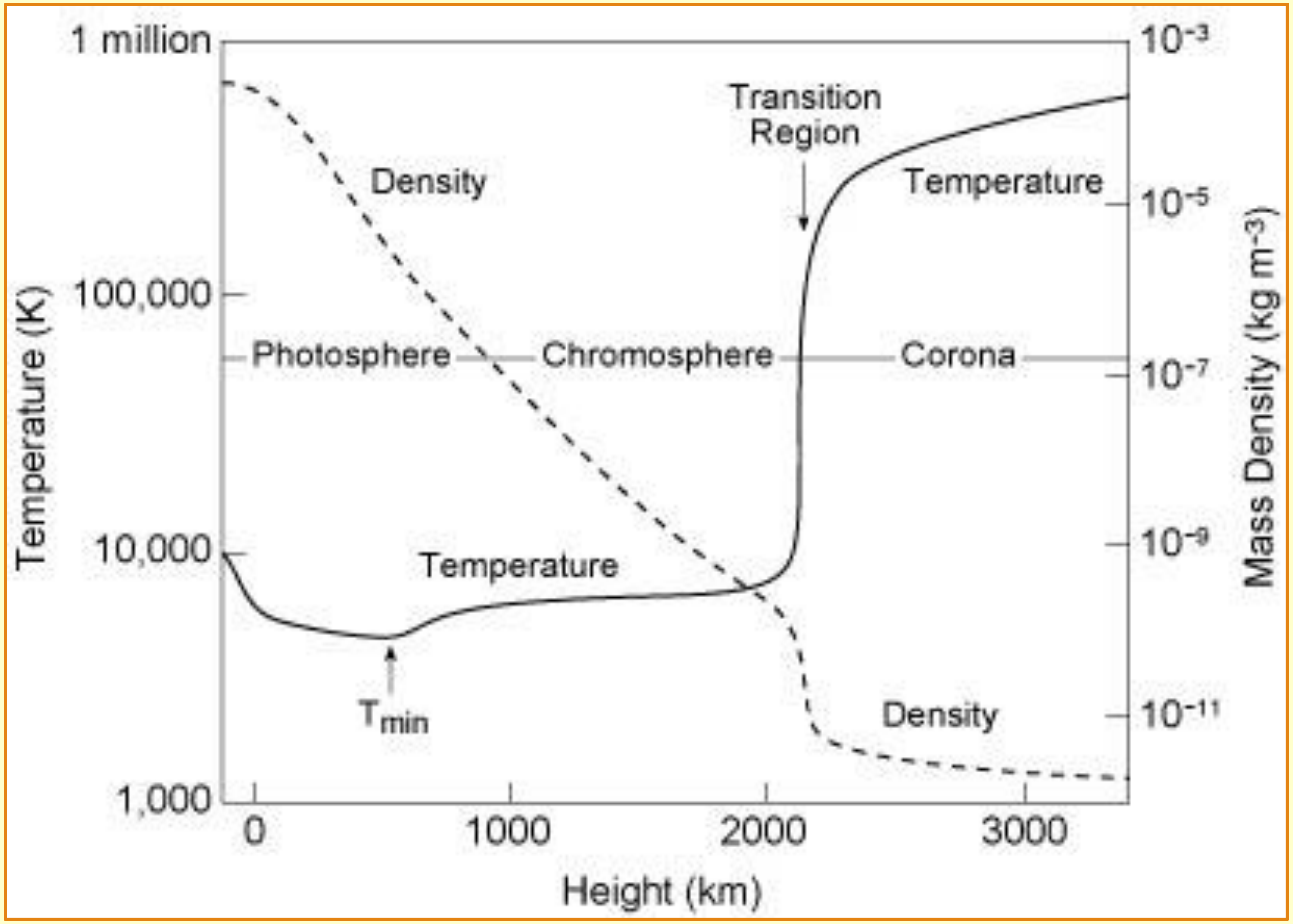
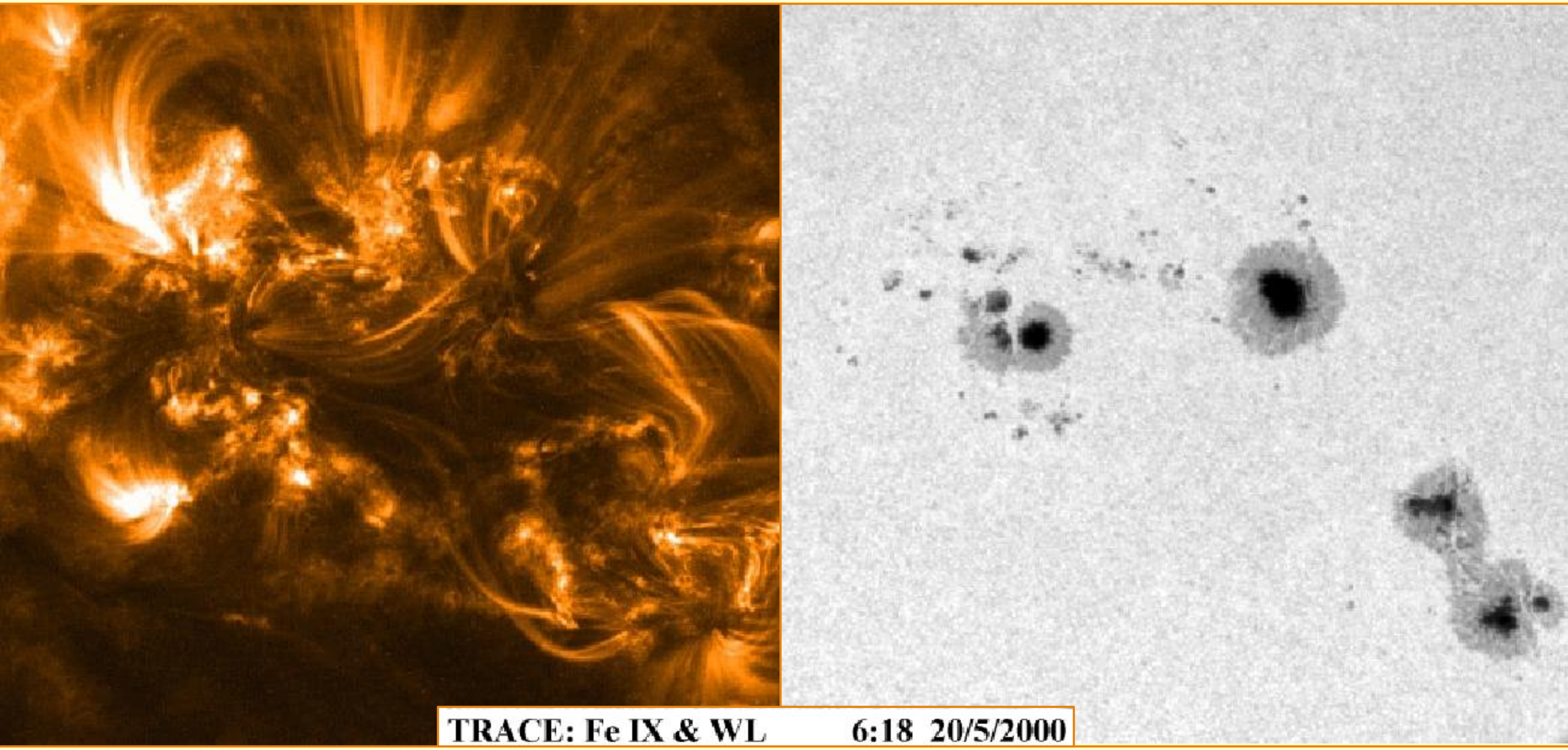
Solar Structure

- Photosphere
  - 5,800k , 10^{-9}g/cm^3
- Chromosphere
  - 40,000k , 10^{-12}g/cm^3
- **Transition Region**
- Corona
  - 3,000,000k , 10^{-16}g/cm^3

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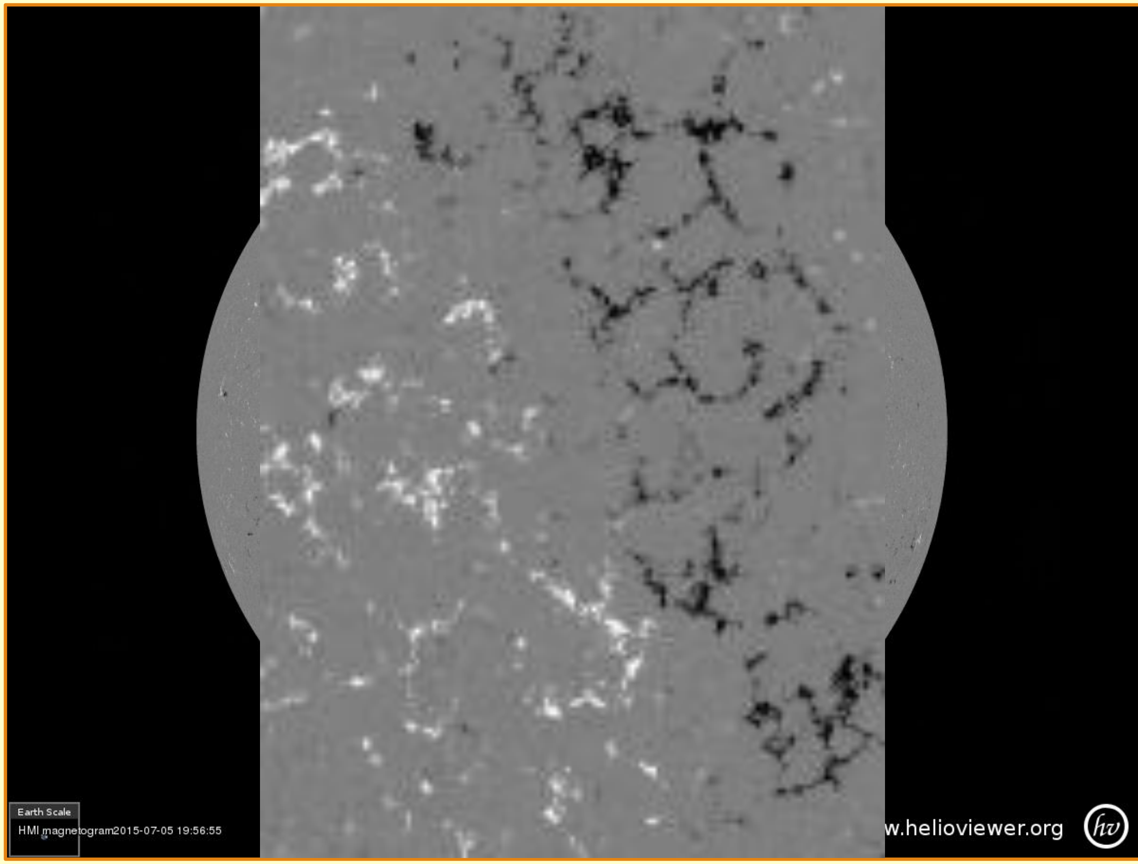
Photosphere

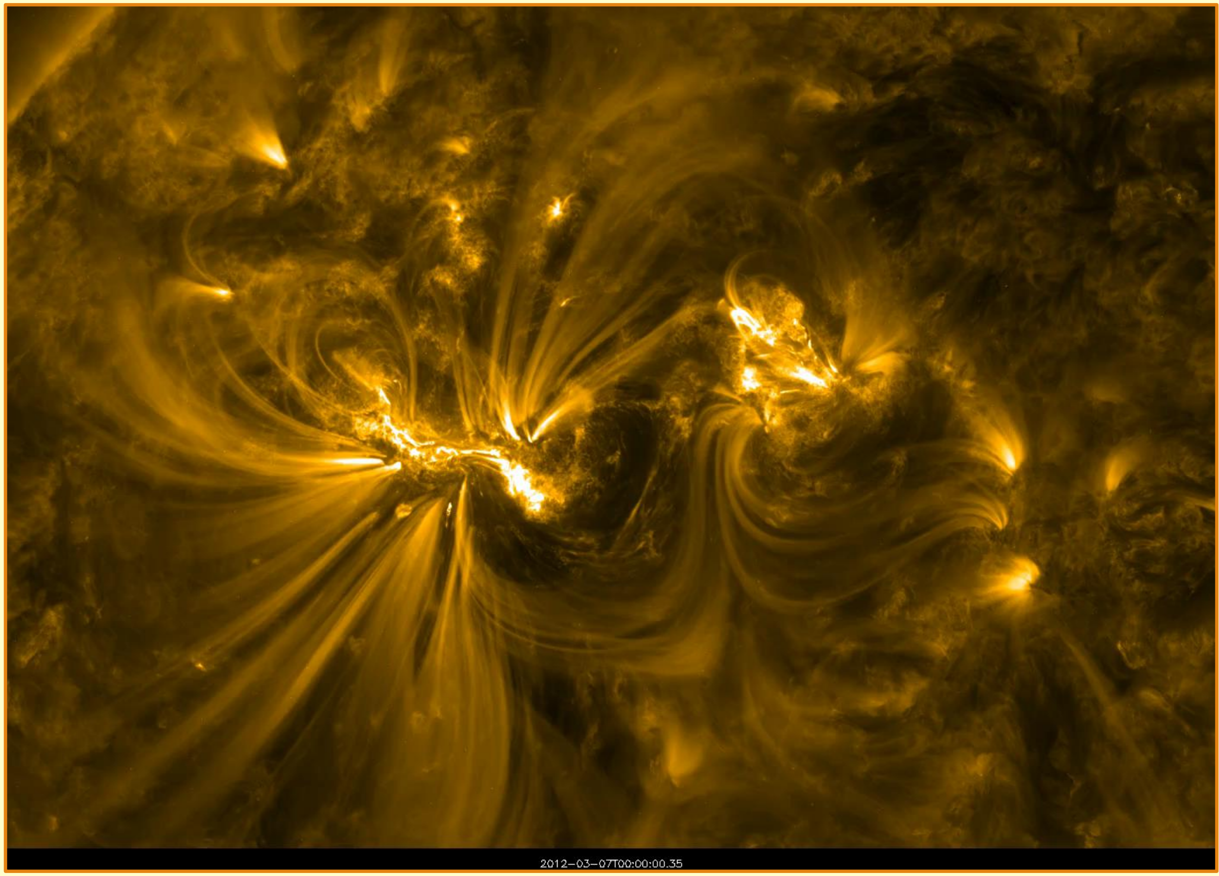
- Activity dominated by B-field
- B-field has foot points in photosphere
- Variety of magnetic properties
  - Granulation
  - Super Granular Cells
  - Ephemeral Regions
  - Active Regions



Corona

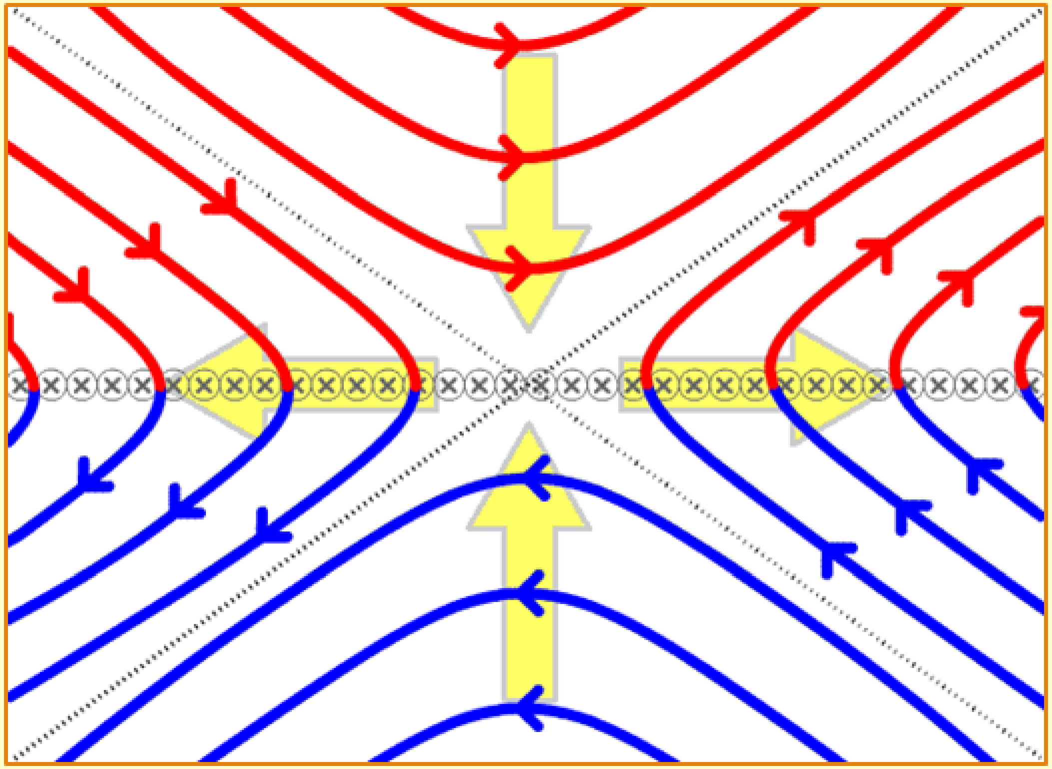
- Two solar flares
  - X-5.4
  - X-1.2

- Post-flare, field lines change connectivity



2D Reconnection

- Well understood
- Restricted, can only occur at X-Type null
- Two flux tubes reconnect
  - break at X-point
  - reform as two new flux tubes
- 2D properties don't transfer to 3D

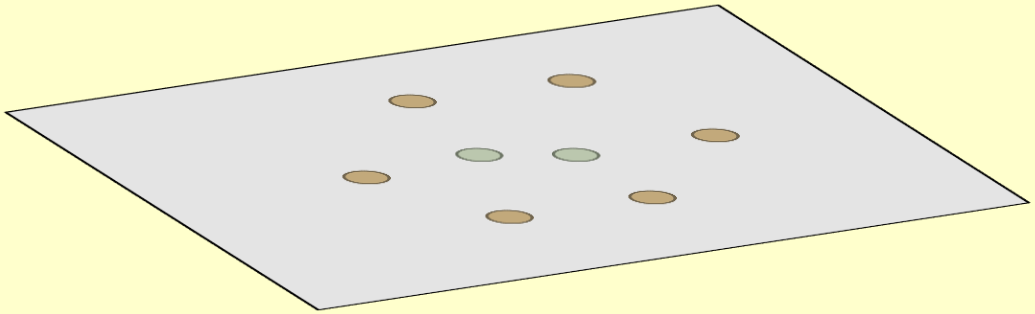


3D Reconnection

- Ongoing field of research
- Not restricted to X-Type nulls
  - Occurs at various topological features
- Flux tubes don’t reform perfectly
- Releases stored magnetic energy
  - Heat, light and particle acceleration

Magnetic Charge Topology

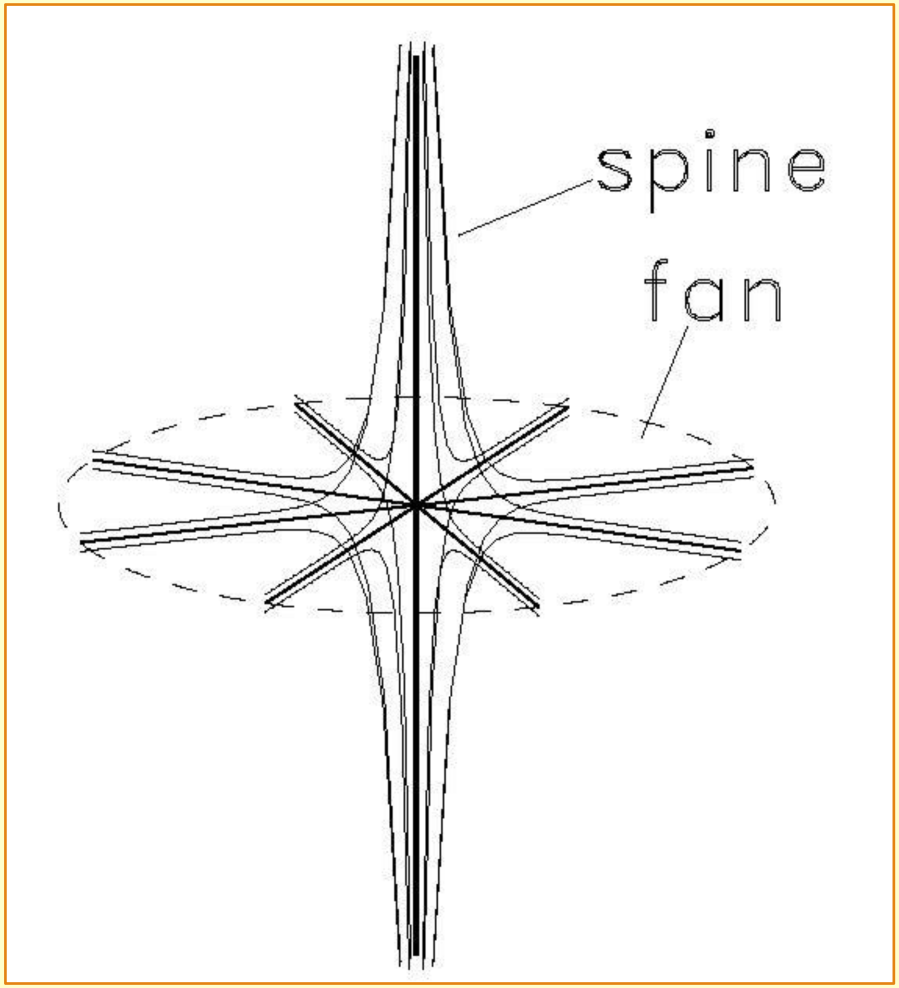
- Simplest, useful model
- Best model for structural simulations
- Assumes j=0, potential
- Treat photosphere as z=0 plane
- Scatter sources of flux on plane



$$\boldsymbol{B}(\boldsymbol{r}) = \sum_i \epsilon_i \frac{\boldsymbol{r}-\boldsymbol{r}_i}{|\boldsymbol{r}-\boldsymbol{r}_i|^3}$$

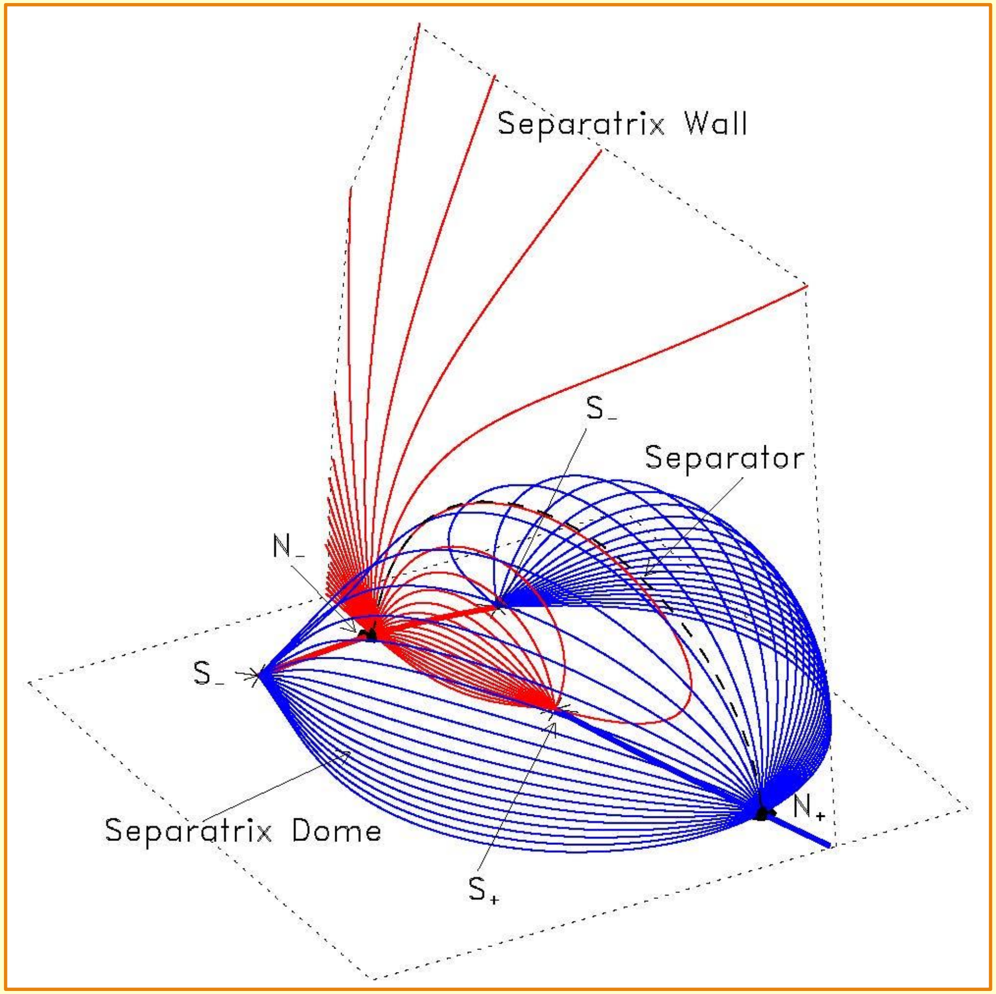
Magnetic Null Points

- We can define null points
  - Locations where $\boldsymbol{B}_x = \boldsymbol{B}_y = \boldsymbol{B}_z = 0$
  - Anywhere in the volume
- Can define spine and fan field lines about a null
- Separatrix surfaces generated from these points
- Intersections in surfaces form separator field lines



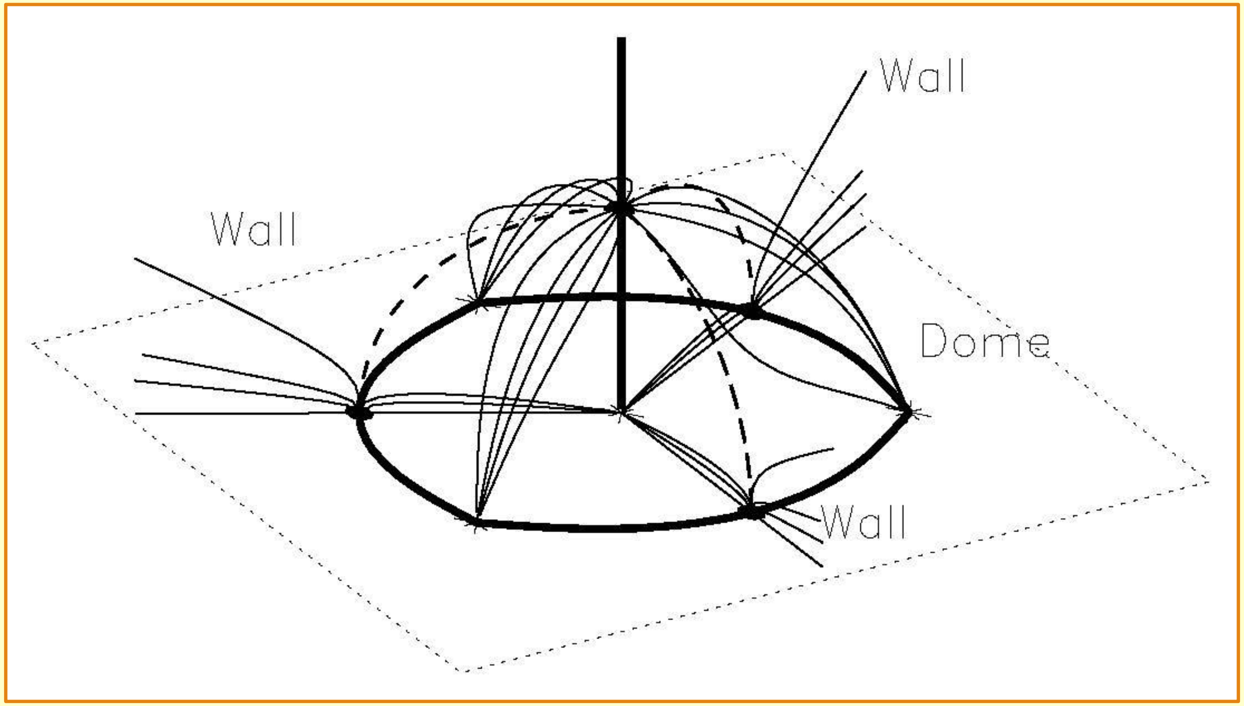
3 Source Cases

- Brown and Priest (1999)
- Various topological structures
- Building blocks for complex cases

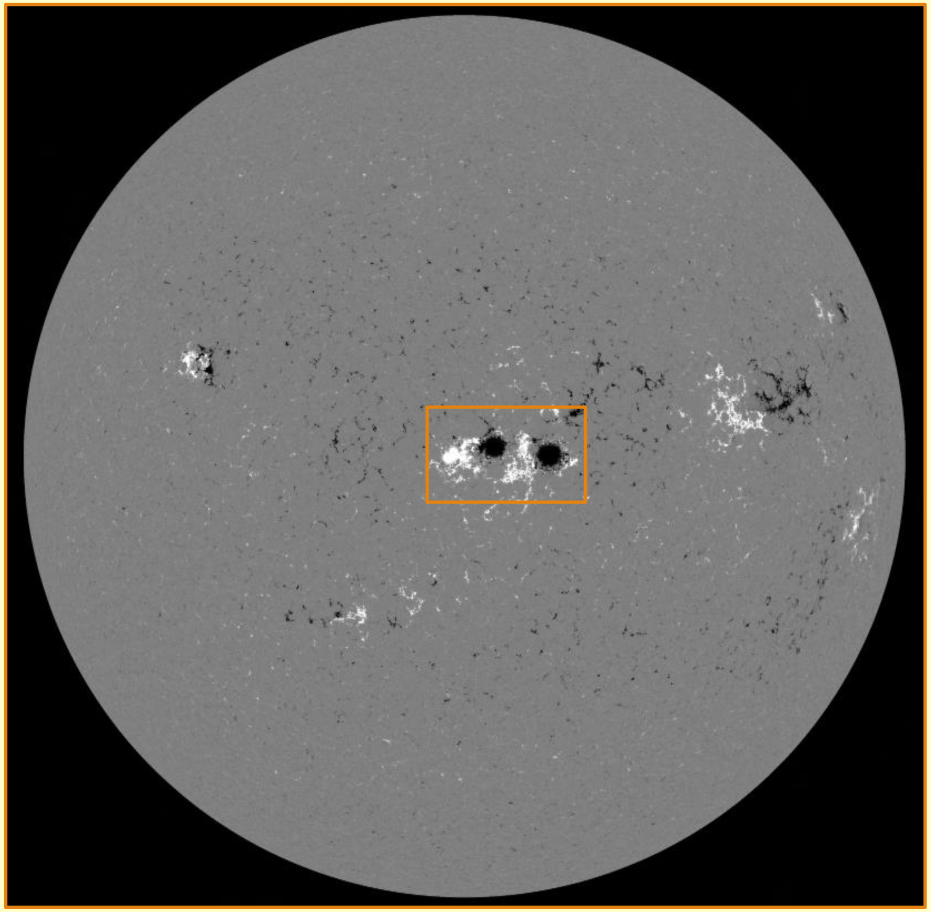


Coronal Nulls

- Null points not restricted to plane
- Flares occur in corona
- Hence want reconnection sites in corona



Source Types



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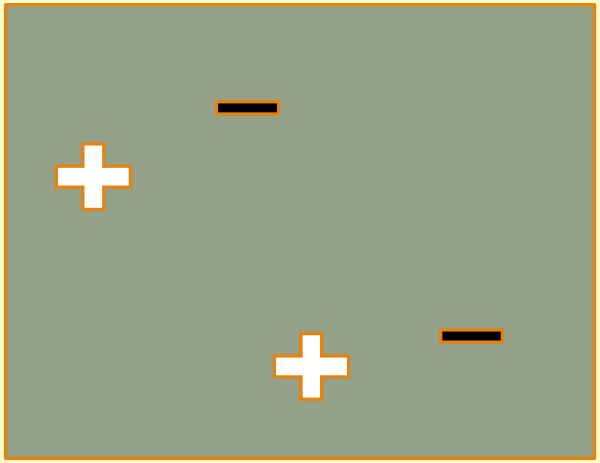
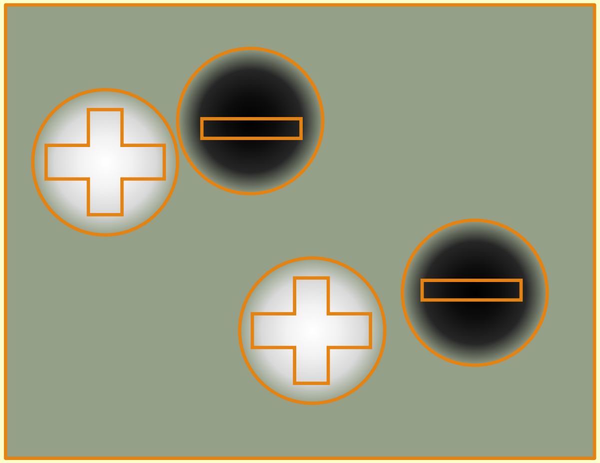
Discrete



Continuous

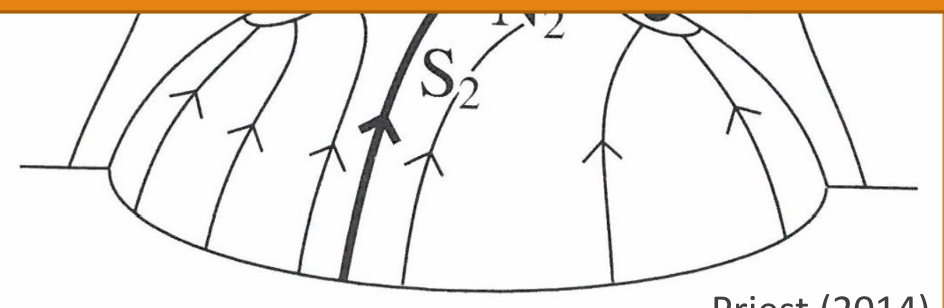


An Open Separatrix Surface

Aims

- Show that topology presented in Priest (2014) may not be complete picture

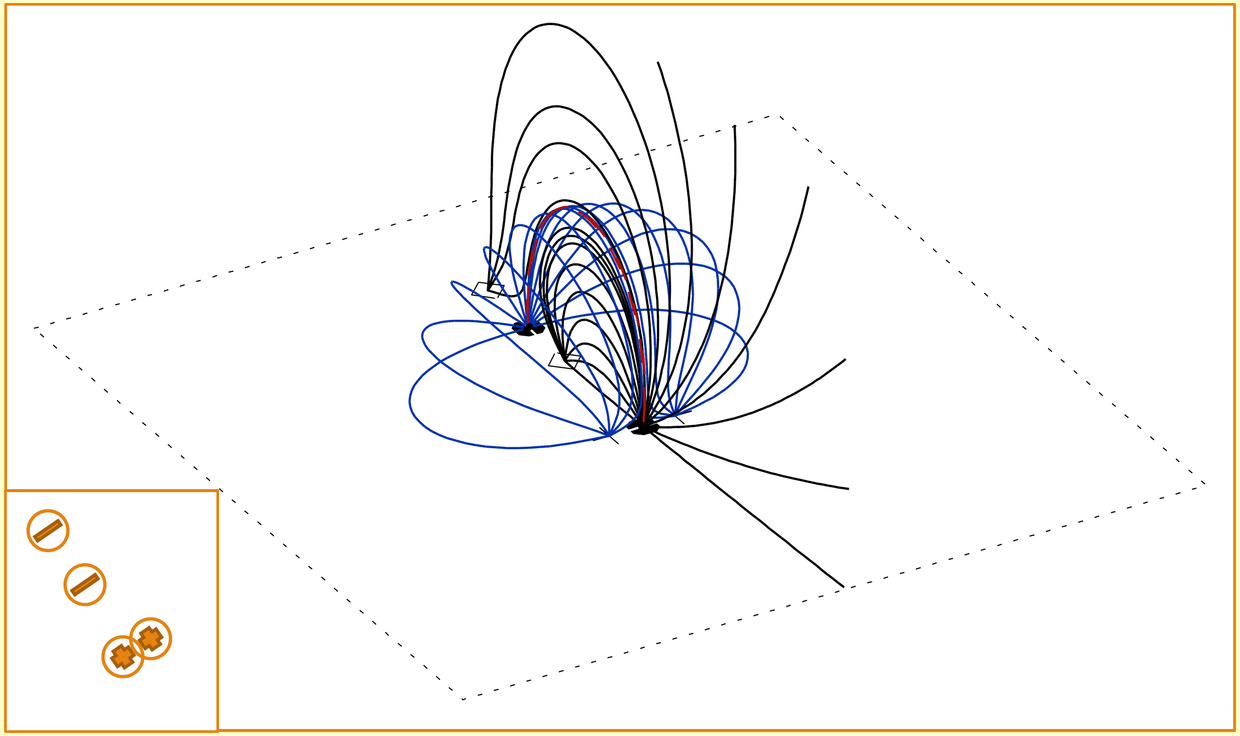
- Define additional features to get more complete picture of topology



Priest (2014)

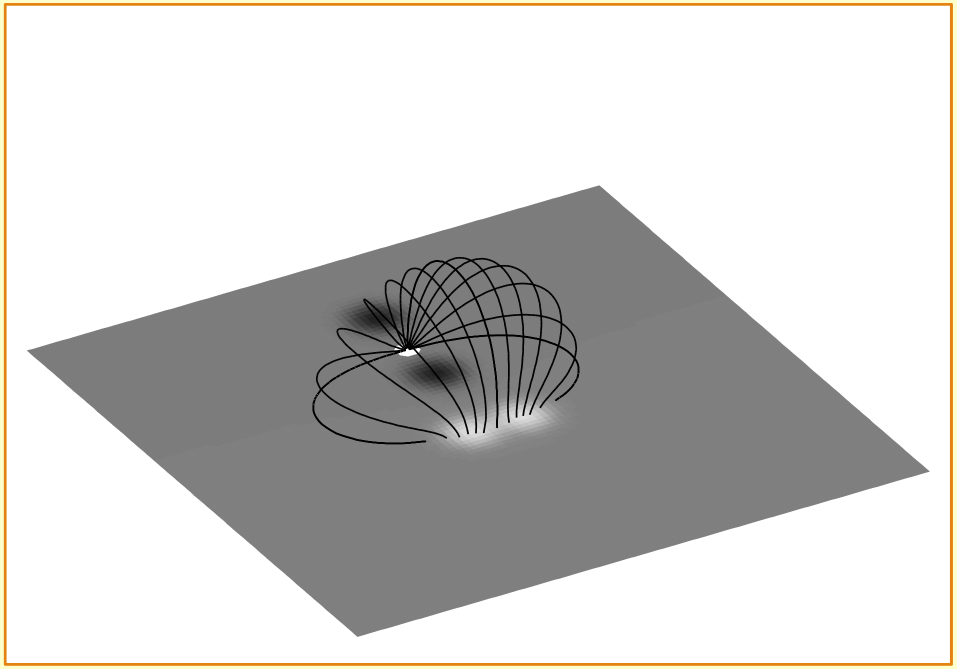
A Discrete Source Study

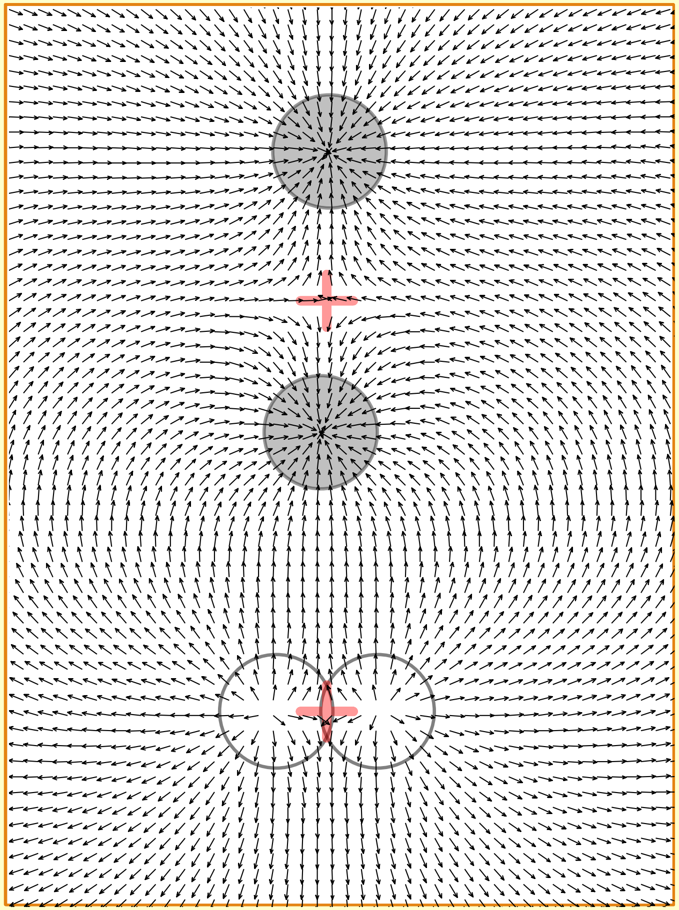
- Produce an intersected state topology with four sources
- Focus on effect moving pairs of sources close together has on topology



A Continuous Source Study

- A continuous source model of same configuration





Null-Like Features

- We can define null-like points
  - Locations where

$$\boldsymbol{B}_x = \boldsymbol{B}_y = 0 \, , \boldsymbol{B}_z \neq 0$$

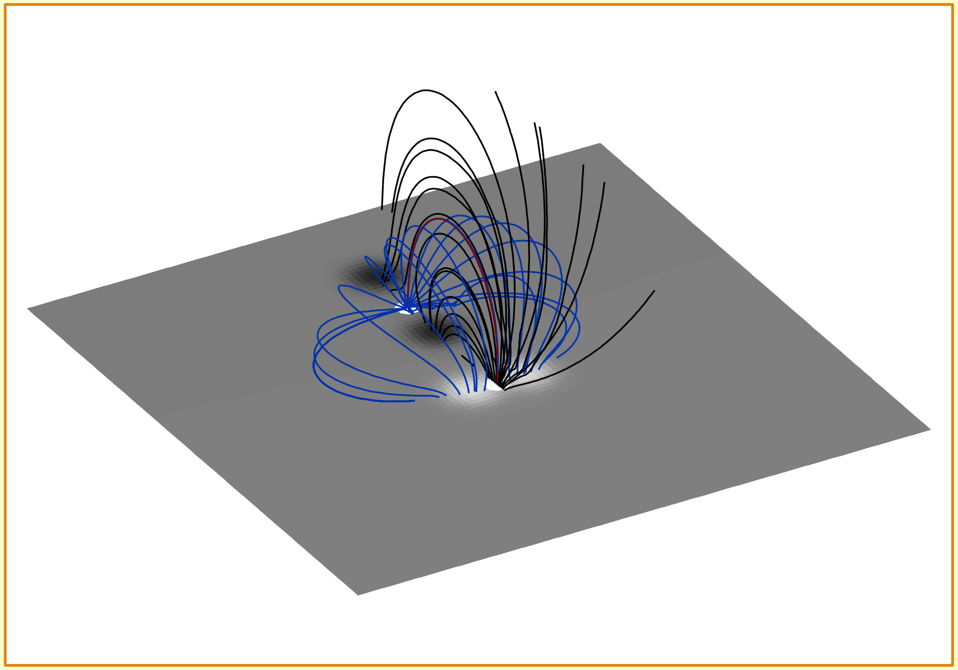
  - Only on z=0 plane
  - Forms an x-line structure

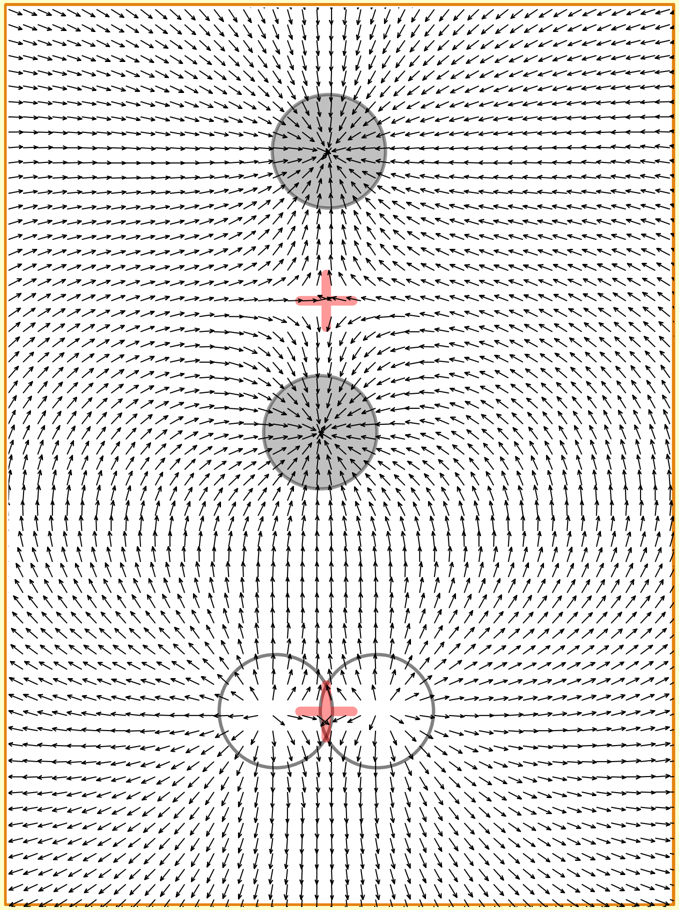
- Separatrix-like surfaces generated from these points

- Intersections in surfaces form separator-like field lines

A Complete Continuous Topology

- Null-like point preserves separatrix wall





Squashing Factor

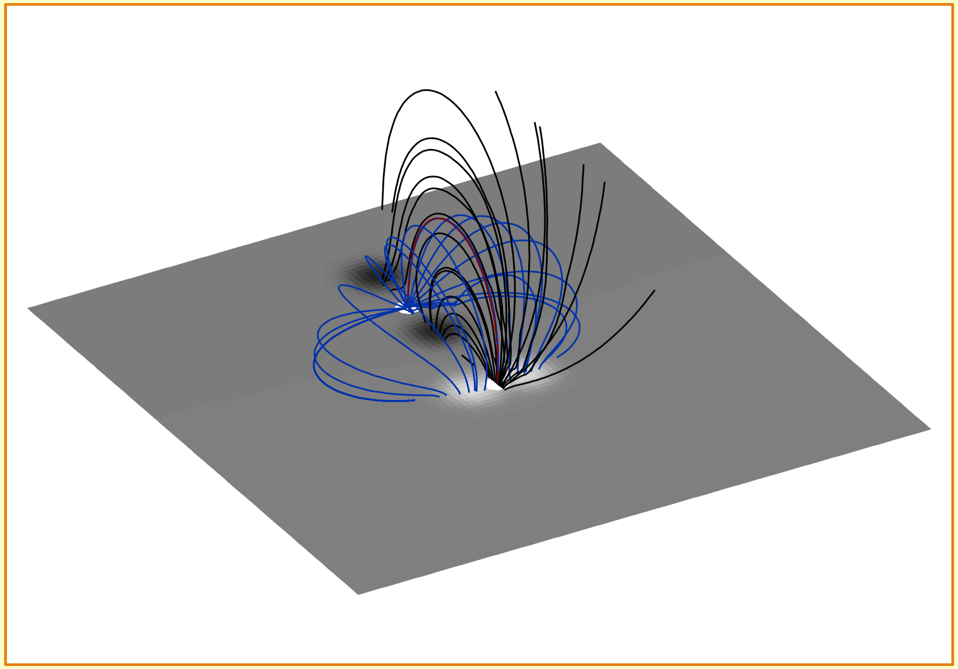
- Squashing factor, Q, provides a measure of field line divergence

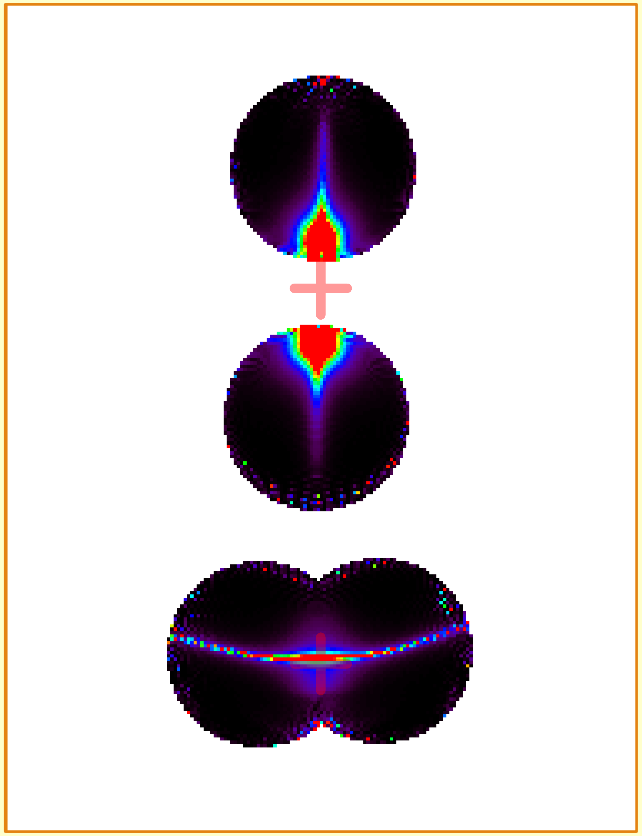
- Initially used as a means of finding quasi-separatrix layers

- Q>>2 implies significant divergence

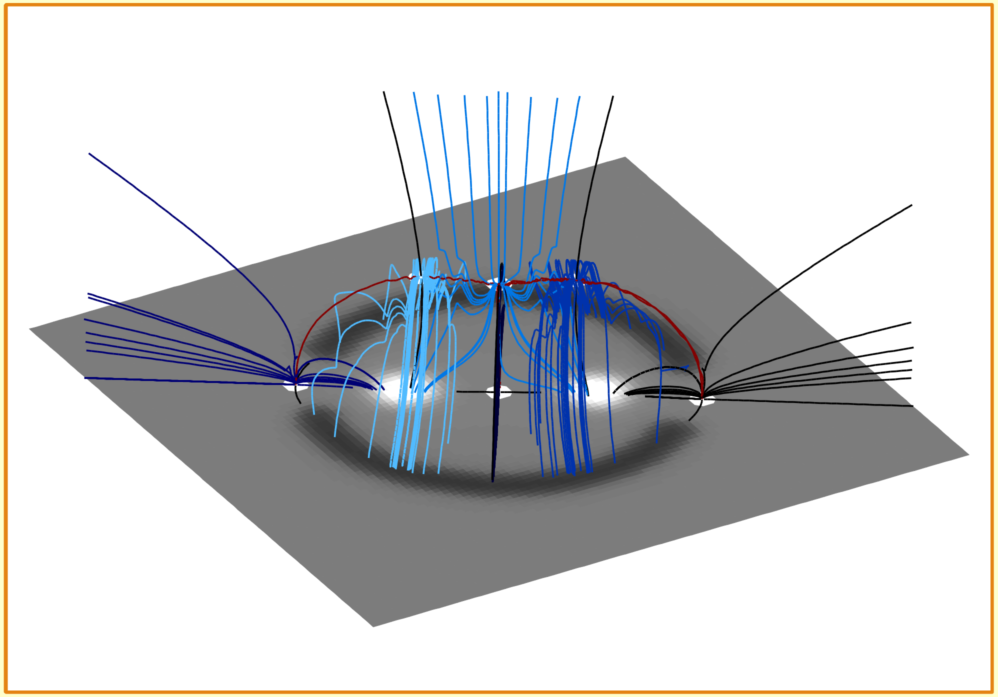
Squashing Factor

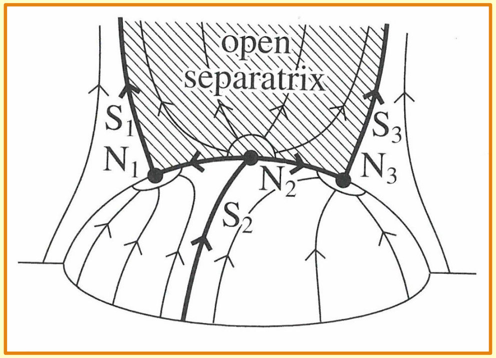
- High Q indicates the presence of topological features





An Open Separatrix Surface





Priest (2014)

Conclusions

Consideration of null-like points is required for a complete picture of a topology

- For Priest (2014) case, inclusion of null-like points suggests open separatrix may not be as open as previously thought

Squashing Factor: Maths

We can define a mapping, Π, which maps from r_+ to r_-.

- $\Pi_{+\,-} : r_+ \rightarrow r_-$

Locally this mapping can be defined by the Jacobian Matrix.

- $D = \begin{pmatrix} \frac{\partial X_-}{\partial x_+} & \frac{\partial X_-}{\partial y_+} \\ \frac{\partial Y_-}{\partial x_+} & \frac{\partial Y_-}{\partial y_+} \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

The determinant of the Jacobian is then used in the calculation of Q.

- $\Delta = ad - bc \equiv \det(D)$

Squashing Factor: Maths

The Squashing Factor, Q, can then be calculated for each cell in a 2D slice of the volume.

- $D = \begin{pmatrix} \frac{\partial X_-}{\partial x_+} & \frac{\partial X_-}{\partial y_+} \\ \frac{\partial Y_-}{\partial x_+} & \frac{\partial Y_-}{\partial y_+} \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- $\Delta = ad - bc \equiv \det(D)$
- $N \equiv \sqrt{(a^2 + b^2 + c^2 + d^2)}$
- $Q = \frac{N^2}{|\Delta|}$